Objectives

> Determine if two events are independent

> Find probabilities using the Multiplication Rule for Dependent Events

> Find probabilities using the Multiplication Rule for Probability

> Learn how to find conditional probabilities

Nov 16-8:49 AM

Probability Rules: Independence, Multiplication Rules, and Conditional Probability

Conditional Probability

- Researchers often want to examine a limited portion of the sample space. For example, consider the question of whether cigarette smoking harms those that are indirectly exposed to the smoke.
- Suppose that 3 % of women who do not smoke die of cancer.
- However, if a nonsmoking woman is married to a smoking husband (not to be confused with a husband who is on fire), the probability of dying of cancer is 0.08.
- This probability is a conditional probability, because the sample space is being limited by some condition—in this case, limited to only wives of smoking husbands. In this instance, the dramatic effect of a smoking husband on cancer rates is readily evident.

P(a nonsmoking woman dies of cancer)≠P(a nonsmoking woman dies of cancer given that her husband smokes)

Nov 16-8:50 AM

- Similarly,
- the results from a market survey indicate that 39 percent of the customers surveyed believe a product is of high quality.
- However, if the analysis is limited to only women, 54 percent of women surveyed believe the product is of high quality.
- Based on the survey, it appears that women have a much higher regard for the company's product than men.
- The difference in attitude would probably be something that could affect how the company spends its marketing dollars.

Nov 16-8:53 AM

Definition

The probability that one event will occur given that some other event has occurred is a **conditional probability**.

The conditional probability of A, given that B has occurred, is

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}.$$

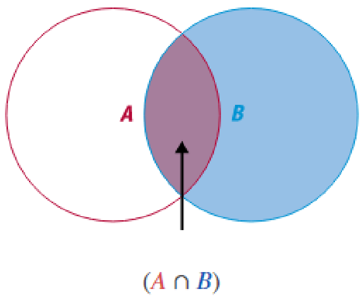


Figure 6.10

The notation P(A|B) is read as The probability of A given the occurrence of B. The vertical bar within a probability statement will always mean given.

Nov 16-8:57 AM

Let's Make a Deal

A long time ago, back in the 70s, there was a television show called *Let's Make a Deal* starring Monty Hall as the host. A new version of the show premiered in 2009 starring Wayne Brady as the host. The Monty Hall show produced an interesting problem in probability that someone submitted to Marilyn vos Savant, which she answered in her column in *Parade* magazine. Incidentally, Ms. Savant was in the Guinness Book of World Records as having the highest recorded IQ (228). Here's the problem that was posed to Ms. Savant.

"Suppose you're on a game show, and you're given a choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say number 1, and the host, who knows what's behind the doors, opens another door, say number 3, which has a goat. He then says to you, 'Do you want to pick door number 2?' Is it to your advantage to take the switch?" Marilyn vos Savant answered the question in her column saying that it was to your advantage to switch. This set off a firestorm of mail telling Ms. Savant that she was incorrect. Much of this mail came from people with Ph.D.s behind their names. The *New York Times* printed a front page article in 1991 discussing the problem.

What do you think? To find the answer to this problem, type "The Monty Hall problem" in your search engine, go to some of the web sites, and try some of the simulations.

Nov 16-9:00 AM

https://www.youtube.com/watch?v=T5QYTrDReTo

Nov 16-9:00 AM

Example 6.9

Suppose a candidate has surveyed his prospective constituents and produced the cross tabulation:

Table 6.2 – Candidate Survey				
Age	Favors the Candidate	Not Favor	Undecided	Total
18–34	213	197	103	513
35–50	193	184	67	444
Over 50	144	219	83	446
Total	550	600	253	1403

If an individual is between 35 and 50 years old, what is the probability he or she will favor the candidate?

Nov 16-9:01 AM

Table 6.2 – Candidate Survey				
Age	Favors the Candidate	Not Favor	Undecided	Total
18–34	213	197	103	513
35–50	193	184	67	444
Over 50	144	219	83	446
Total	550	600	253	1403

If an individual is between 35 and 50 years old, what is the probability he or she will favor the candidate?

Let the events

$A = \{\text{favor the candidate}\}$,

and

$B = \{\text{age between 35 and 50}\}$.

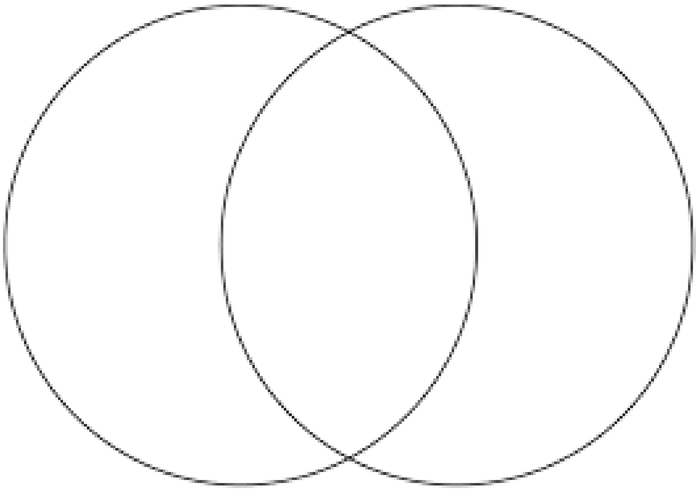
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}.$$

$$P(A \cap B) = \frac{193}{1403} \approx 0.1376 \quad P(B) = \frac{444}{1403} \approx 0.3165$$

$$P(A \mid B) \approx \frac{0.1376}{0.3165} \approx 0.4348.$$

Nov 16-9:04 AM

Table 6.2 – Candidate Survey				
Age	Favors the Candidate	Not Favor	Undecided	Total
18–34	213	197	103	513
35–50	193	184	67	444
Over 50	144	219	83	446
Total	550	600	253	1403



Nov 16-9:08 AM

Independence

An extremely important concept in statistical analysis is independence. It describes a special kind of relationship between two events. Two events are said to be independent if knowledge of one event does not provide information of the other event's occurrence. In other words, the occurrence of one event does not affect the occurrence of another event if the events are independent.

Nov 16-9:09 AM

Basketball and Dependence

- If you were playing basketball and made a large number of consecutive shots, then there might be a temptation to boast of your skills.
- Alternatively, someone might suggest that you were on a lucky streak.
- Two psychologists examined the "lucky streak" phenomenon by analyzing the sequence of made and missed shots by professional basketball players.
- The selected players made roughly 50 percent of their shots.
- Their analysis found that there was no evidence of the hot hand—that is, there was no evidence of a dependent relationship between the consecutive shots.
- They did not find more long streaks of made baskets than would be expected to occur by chance.

Nov 16-9:09 AM

Example 6.10

Experiment: roll a fair die two times. Consider the two events

$A = \{$rolling a six on the first roll of a fair die$\}$ and

$B = \{$rolling a four on the second roll of a fair die$\}$.

Are these two events independent?

Solution

Since knowledge of the outcome of the first roll does not help one make an inference of the outcome of the second roll, events A and B are independent.

Nov 16-9:11 AM

Definition

Two events, A and B, are **independent** if and only if

$$P(A \mid B) = P(A) \text{ and } P(B \mid A) = P(B).$$

Nov 16-9:11 AM

In many cases, regarding the independence of two events, intuition and common sense will lead you to the correct determination. However, there are situations in which independence can only be discovered by formal application of the definition.

What does it mean if two events are not independent? The obvious response is to say that they are dependent, a term that is just as much a part of statistical vocabulary as independent. If events are dependent, they are related; the nature of the relationship and whether the relationship can be used for predictive purposes are problems often examined by statisticians.

Nov 16-9:11 AM

During the course of a business negotiation, both negotiators may exhibit numerous types of idiosyncratic behavior. If they have jewelry, they might manipulate it. If they smoke cigarettes, they might play with their lighters or packs of cigarettes. Are their mannerisms independent of the importance of the issue they are negotiating? Does the negotiator tend to smoke a cigarette or play with jewelry when he or she has a strong position? Good negotiators will pick up dependencies and use the information to their advantage. However, the concept of association implied by dependence must not be confused with the idea of causation. It may be that one of the events does indeed cause the other, but the fact that they are not independent (dependent) is not evidence of causation.

Now let's consider the probability of two events both happening. For example, what is the probability of choosing a queen and then a king from a deck of cards? The key word in these problems is the word "and".

Nov 16-9:12 AM

Before we can calculate these types of probabilities, we need to introduce a few new terms. The phrase with repetition means that outcomes may be repeated. For example, if you are choosing numbers with repetition for a bank pin number then you are allowed to repeat numbers, such as 1231. Similarly the phrase with replacement refers to placing objects back into consideration, such as choosing a card from a deck and then returning it to the deck for the next choice. On the other hand, the phrase without repetition means that outcomes may not be repeated. For example, if you are choosing a bank pin without repetition then you are not allowed to repeat numbers, such as in the pin 1234. In the same manner, the phrase without replacement means your first choice is not put back in for consideration, such as drawing a card and then drawing a second card from those left over. These four phrases affect the number of possible outcomes in the sample space.

Nov 16-9:13 AM

Lastly, two events are independent if one event happening does not influence the probability of the other event happening. For example, if after drawing a card you replace the card drawn–and shuffle the deck–then the probability of the next card drawn is not affected by what was picked first; therefore, choosing the two cards from a deck with replacement are independent events.

Nov 16-9:21 AM

Probability Law 10: Multiplication Rule for Independent Events

If two events, A and B, are independent, then

$$P(A \cap B) = P(A)P(B).$$

If n events, $A_1, A_2, \ldots, A_n$, are independent, then

$$P(A_1 \cap A_2 \cap \cdots \cap A_n) = P(A_1)P(A_2)\cdots P(A_n).$$

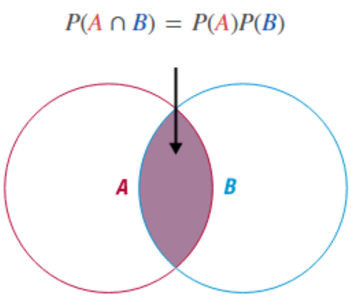


Figure 6.11

The multiplication rule for independent events (Probability Law 10) is sometimes called the product rule. The rule simply states that the probability of the joint occurrence of independent events is the product of their probabilities.

Nov 16-9:22 AM

Let's look at a few examples of calculating probabilities using the phrases *with replacement* and *without replacement*.

Say we choose two cards from a standard deck, with replacement. What is the probability of choosing a two and then a five? Because the cards are replaced after each draw, the two events are independent. Using the Multiplication Rule for Independent Events, we find

$$\begin{aligned} P(\text{two and five}) &= P(\text{two})P(\text{five}) \\ &= \left(\frac{4}{52}\right)\left(\frac{4}{52}\right) \\ &\approx 0.0059. \end{aligned}$$

Nov 16-9:25 AM

Example 6.11

A coin is flipped, a die is rolled, and a card is drawn from a deck of 52 cards. Find the probability of getting tails on the coin, a five on the die, and a Jack of clubs from the deck of cards.

Solution

Since the three events (flipping a coin, rolling a die, and selecting a card) are independent, we can use the product rule.

We know the following.

$$P(\text{tails on coin}) = \frac{1}{2},$$

$$P(\text{five on die}) = \frac{1}{6}, \text{ and}$$

$$P(\text{Jack of clubs}) = \frac{1}{52}.$$

Therefore, we can calculate the probability as follows using the product rule.

$$\begin{aligned} P(\text{tails on coin} \cap \text{five on die} \cap \text{Jack of clubs}) &= P(\text{tails on coin})P(\text{five on die})P(\text{Jack of clubs} \\ &= \left(\tfrac{1}{2}\right)\left(\tfrac{1}{6}\right)\left(\tfrac{1}{52}\right) \\ &= \tfrac{1}{624} \\ &\approx 0.0016. \end{aligned}$$

So the probability of getting tails on the coin, rolling a five on the die, and then selecting the Jack of clubs from the deck of cards is approximately 0.0016.

Nov 16-9:29 AM